A PROGRESSION IN Arithmetical Progresſion,

To the ſpeedy ſuming up of any Totals of Roots, Squares, or Cubes as effectually as now is uſed only in Numbers, with various uſes thereof.

WITH

Some things very remarkable in that Myſtical Number, 666.

ALSO

Affording eaſe in the Extracting of Cubick Roots; and the framing of Cubes to imperfect Roots.

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By J. W. of *Brandon* in *Com' Warwick*, Gent.

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THE EPISTLE TO THE READER.

Beloved Reader,

THE *Lord who is a God of Knowledge, and in whom are hid all Treasures of Wisdom and Knowledge, hath declared in his Word,* Isa. 11. 9. That there shall be no hurting nor destroying in all his holy Mountain, for the knowledge of the Lord shall cover the Earth, as the Waters cover the Sea; *which though it be (doubtless) principally meant of such a saving knowledge of him as leads men into a conformity to his Will (seeing* they that say, They know him, and keep not his Commandments, are Lyars, 1 *John* 2. 4.) *and Sons of* Belial, *though Priests are said not to know the Lord,* 1 Sam. 2. 12. and they that handle the Law, *Jer.* 2. 8.

Yet, being spoken indefinitely, it doth not ex-

clude natural knowledge, which the Lord in his infinite Wisdom hath reserved much of it to be communicated to the World for the fulfilling of his gracious promise. Of how late a Date is the knowledge of the Load-stones to direct in Navigation? and how little hath the World rendred to the Lord, according to that benefit received: but rather it hath been abused by too many as a means to further Pyracy and War.

How hath the Lord incited many eminent Persons to the search after an increase of Natural Knowledge in this our Age; and hath succeeded in their endeavours in producing some worthy things; but their Speculations have been elevated according to their Educations to more sublime Orbes, overlooking this inferior Orb, whence more profitable things, and less curious, may, by improving of rational Maxims be deduced to a greater advantage to the World, which once being advanced to a higher Orb, especially nigh to the Transcendent Luminary therein, may by the Beams derived therefrom be rendred more conspicuous to the whole World in their circular motion thereabout: as in that admirable Comet lately did appear, which being but mean in appearance, in respect of its own quality; yet by reason of its vicinity to the Sun, was made splendid to no small admiration, who then should despise the day of small things: Behold how great a matter a little fire kindleth. This spark of an increase of Knowledge in a Science that hath been accounted of

so little use (as the Author mentioned in the beginning of the Treatise following doth write) may inflame others, to that so divine and exquisite knowledge (as he writes of the Rules of Algebra, *then which he saith he never saw any thing amongst the Mathematical Arts, more excellent and elegant; and possibly that which hath rendred Arithmetical Progression more useful and extracting of Cubick Root more easie, may have some tendency towards the illustrating and facilitating of the other also, the knowledge of Square and Cubick Root conducing thereto) for my own particular I never read any thing of that nature, but what is hinted at in the foresaid Author, mentioned in the close of this small Tract; he writes in* Page 45, *of his Book of Arithmetick, that he before all others had improved the Rule of Falshood to resolve many Questions of the first, second, third, and fourth Rules of* Algebra, *which another writ was impossible to do; and this to be effected by Multiplying into Squares and Cubes, and higher Degrees, as also by extracting of all sorts of Roots; so may I safely affirm, that what I offer was never before shewed,* viz. *so great a use of Arithmetical Progression, the depth whereof I possibly do not understand.*

As also I may further say, that none hath bid so fair for a probable interpretation of that mystical number 666 before me.

From the consideration whereof, a light to this

advancement of knowledge in Arithmetical Progression did first arise; for finding first the Digit 6 to be a total of Roots: it put me upon search after 66, and finding that the like, I was lead to an enquiry after 666, and found that to be a Total also, but 6666 is not so.

Which 3 degrees of sixes, though they agree in being Totals of Roots, yet the two lesser Totals differ in form from the greatest 666; and each is a very imperfect number, with respect to Squares and Cubes, which are of all men acknowledged to be very perfect and Stable, having a firm Basis or Foundation to each of them, as in this small Treatise will appear, and of how great use the Digit 6 is in the Cube numbers, on various considerations.

The knowledge of these things being of general Concernment to the World, and imparteth freely have received it: I shall not so far distrust its acceptance with the Reader of greatest Accompt, as to make an Apology for the same; or for any defect in exhibiting the same in no more eloquent expressions, matters of this concern not so much requiring elaborate expression, as Plains in Demonstration.

Moreover, Hereby I signifie to the World, that as I offer this thereto for the increase of knowledge therein, so I design the seconding of it with the improvement I have experienced of another Art to a more Universal Good to all both rich and poor, viz. An Appendix to *Markham*'s Art of Fowling, ready for the Press a year since.

A PROGRESSION IN Arithmetical Progreſſion.

Common uſe of Progreſſion Arithmetical.

ONE Author treating of Progreſſion Arithmetical (whoſe name is inſerted in the cloſe of this Book) *page* 20. writes that he knew no uſe of it: But a compendious Addition, and no other (that I know of) aſſigns it to any other end, but to ſum up Roots or Numbers in the enſuing way; as to ſum up 1, 2, 3, 4, 5, 6, 7, 8. 9, 10. add alwaies the firſt and laſt ſums together, which here make 11. and then divide by 5 (the half number of places, and it gives 55 total to the ten firſt Roots, *&c.* but if the number of places be odd, as 1, 2, 3, 4, 5, 6, 7, 8, 9. then the firſt and laſt numbers added will be even, and the number muſt be me-

mediated, and product multiplied by all the places: as here 9 and 1 make 10, which equally parted is 5, and that multiplied by 9, is 45 total to the first 9 Roots.

Now whether the Progression begin at a Unite or no, or whether they succeed by more than one, as 1, 3, 5, 7, 9, 11. or 3, 6, 9, 12, 15. and the like equal differences: the way of summing up, is still the same as before. Now before I proceed to shew other uses of it, which are various, I shall briefly mind what is remarkable in this known way of Progression, not observed in any Author that I have seen.

Of framing Squares by Progression Arithmetical.

First, As Progression by Unites from a Unite makes a total: so Progression from a Unite in odd numbers each makes a Square, so 1, 3, make four square to 2; and 1, 3, 5, make 9, the next Square, and so on; and Progression from 2 or by 2 in even numbers, gives the Square and Root United, half of which is alwaies the total: so 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, give 110, and 9 such Progressions gives 90, which is 81, and 9, the other 100 Square and 10 Root.

Of

Of framing Cubes by Progression; and some Hints of Extracting higher Roots; see more *p.* 13.

Now a further use of this known way of Progression (unobserved) is to constitute Cubes, which usually are framed by Progression Geometrical, which is a proceeding by doubling, trebling, or the like of any number, as 2, 4, 8. 3, 9, 27. 4, 16, 64, *&c.* which would be observed by any that shall attempt the facilitating the Extraction of higher Roots than Square and Cubick: which I am ignorant of the use of and skill in, having never seen any thing leading thereto, but some fragments of a Book in Quarto, styled, *Masterſons Arithmetick*; in which was a Progression by prodigious numbers to *Zenze Cube ſur ſolid.*

As also I have seen some more than ordinarily use of it hinted at in the Author mentioned in the close, *viz.* to resolve some sublime Questions pertaining to the Rule of *Algebra.*

How to frame Cubes by Arithmetical Progression.

By the ordinary Rule of Falshood, which I shall ground a Petition from in the close, to those that have higher attainments to make the knowledge thereof more common, if it be more useful. But now to manifest how the same thing is effected by Progression Arithmetical, which is pro-

produced by Geometrical, as before, it is no otherwise before; only the Root given must be the first Progression, and also the number of places, as if the first be 10, then for ease in the work, say 1 ten, 3 tens, 5 tens, and so proceed 10 places, it brings to 19, which multiplied by 10, gives 190, to which the first 10 being added makes 200, which multiplied by 5 makes 1000 Cube to 10, and so in any other even or odd as before.

There is also a way to frame Cubes by Progression Arithmetical from a unite, but the knowledge of it requires skill in extracting square Root, and therefore it is referred to another place in this Book.

Before I proceed to hold out other necessary uses of Progression Arithmetical, it is requisite for those that are ignorant to define what a Cube is.

What a Cube is.

Now a Cube is a Body whose 3 dimensions of length, breadth, and height are exactly equal; it hath 6 Superficies, and 8 Angles, and is fitly represented by a Dice, whence the Proverbs arising, *As firm and as square as a Dice*, denote a Cube to be a perfect number, and an emblem of sincere Christians, for it is constituted all of squares: Now look how many unites the square contains, so many squares doth every Cube contain: for instance,

As 12 times 12 make 144 its ſquare, ſo 12 times 144 make 1728 Cube to 12 Root.

How to Meaſure Board.

A Board every way 12 inches is ſuch a ſquare, and 12 ſuch ſquares inch thick, are a foot of Timber, and 144 is the Standard to meaſure boards by, as alſo 1728 is the ſame for Timber, ſay by the backer Rule of 3 if 12 inches board in breadth give a foot or 12 inches in length, what will 8 inches in breadth give: now in this Rule, the two firſt numbers being to be multiplied together, the dividend will be alwaies 144, and the laſt ſum the diviſer, which here being 8, gives 18 inches in length to a foot of board whoſe breadth is but 8 inches.

How to meaſure Timber.

So 1728 will alwaies be the dividend when any ſquare of Timber is required, to know how much thereof makes a foot in length.

As if 144 ſquare give 12 inches in length to a foot, what will 72 give? that is 8 inches one ſide, 9 the other; divide 1728 by 72, and it gives 24 inches or 2 foot, and this manifeſts how neceſſary the knowledge of Cube numbers is.

Dignity of Cubes.

But beſides the neceſſity thereof, this figure of a Cube hath a great dignity in it, being choſen by the Lord, to be typical of Chriſt and his Church; he preſcribed his incenſe Altar, to be in a Cu

a Cubical form it consisted of two Identick Cubes united in height, *Exod.* 30. 1. his Holy Oracle wherein he chose to place his name and dwell, was of a Cubick figure, 120 Cubits in length, 20 Cubits in breadth, and 20 Cubits in height, 1 *Kings* 6. 20.

And Type of his Gospel Church God hath chosen *Zion*, and desired it for his Habitation for ever, *Psal.* 132. 13, 14. 68. 16. *Zeph.* 3. 15.

And also a Type of every true believing Christian, 1 *Cor.* 6 19. 2 *Cor.* 6. 16. *Eph.* 2. 21. 1 *Pet.* 2 5.

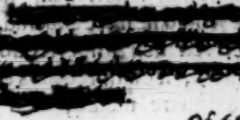
These two last Scriptures speak clearly God's way of Building his Church. Now to the Law and to the Testimony, if they speak not according to this Word; see the reason, *Isa.* 8. 20.

Further, as the Lord dignified this Form to be a Type of his Church: so see it described in this form by him that had the Testimony of Jesus, *Rev.* 21. 16. the length and breadth and height thereof was equal.

See the House which he that was worthy of more glory than *Moses* hath built, *Heb.* 3. 3, 6.

Measure the pattern, but first see what is required, *Ez.* 43: 11. then measure with it the Worshippers in it, *Rev.* 11. 1. it is to be done by a Reed there, and that a Golden one, by which the measure cubical is produced, *Rev.* 21. 15. And behold this is the Law of the House, *Ezek.* 43. 12. Holiness becometh the House of God for ever, but God dwelleth not in Temples made with hands.

Now the Lord having in these and many more Scriptures, described his true Church and People, and the way of building this Tabernacle of *David*, by taking out of the Gentiles, a People to his Name (it is Gods handiwork, *Isa.* 29. 22, 23. 60. 21. 61. 1, 2, 3, 4.



Of 666.

But to return to our matter as a cubical figure, fitly signifieth a true Church and Christian: So as all agree the false Church and Christian is set out in that mystical number *666*, which is the first total of Roots that is presented to us in such a unity of digits. tens and hundreds, and in shew of perfection, as *Bernard* on the *Revelations*, *pag.* 240. observes and recites *Junius* to do the like.

Further he saith, Pope *Boniface* the Eighth, speaketh of *6* as a perfect number digesting his decretals into *6* Volums which he calleth a perfect Volume containing (as the Pope saith) a perfect platform for the Government of affairs, and a perfect Discipline for the Rule of good manners. Thus we see, saith *Bernard*, the high accompt the Pope makes of this number of six, and descanteth farther on the false Church differing in degrees, yet agreeing in uni-

ty to the Pope, and his perfect Decretals.

Of 666 being imperfect in every degree.

And further taking notice of 6 as being but half of God's number, which is 12, and the number of his Apostles, whence he concludes the false Church to be but half Apostolick, half Christian; much more would he have concluded the same if he had perceived such imperfections in it as I have in part hinted, it comprehends but half a Root, and half a square united, and so falleth very much short of a true square in its Superficies, and so is every Digit and degree therein 6·66, and 666 each of them carrying such a proportion to a true square; moreover 666 is but half a Cube, and half a unite over; and the total to that Cube, of which it is half, is 66 to 11. whose Cube is 1331, and 666 doubled gives 1332.

Besides it wants 10 of a just square, which is 676 square to 26, and so hath no perfect square Root; I leave every one to infer hence what they please, and shall note further how that totals, squares, roots, all arise from 1, but proceed in a different form. Totals in a very imperfect form, even totals in one form, odd totals in anothers: but Squares and Cubes proceed alwaies alike; Squares and Cubes above what it may be on total or universality of numbers.

The

The first is 10 total to 4 root, the last 6 total to 3 root. this related to patterns of totals in the

Other use of Progression Arithmetical.

But after so long a digression, I return to shew greater use of Progression Arithmetical; to conceive whereof, suppose the order reversed, as the last to the lowest, and the number of Digits in the root to be so many foot, as 36 foot or 12 yards; the next 35 foot will lie firmly on that being 6 inches narrower on each side, and so forward upward to one; at top it ariseth to 666 foot, that is, 222 yards in height, and so hereby gathered from how narrow a foundation being firm a cubical ascent may be *ad libitum*, it sheweth the whole contents of such cubical Pyramid (supposed to be solid) by multiplying the total in it self; so 666 multiplied by 666, gives the contents of 36, first progressive Cubes.

How to sum up any number of Cubes.

How therefore to find out difference of Cubes.

A farther use of Progression Arithmetical, is, that thereby any that hath skill in Multiplication and Progression, though he know not what a Cube or square is, may easily come to know what the difference of any Cube is from its next precedent or subsequent Cube, for it is no more but to search for the total and multiply it by 6, and there are so many unites betwixt: excluding each Cube, else if you take one Cube out

of another, there will be a Unite more.

So it being required to know the difference of the Cube to 20, from the Cube to 21, seek alwaies the total to the first, which is here 210, that multiplied by 6 makes 1260, now take 8000 Cube to 20 out of 9261, it leaves 1261, and every last Cube contains the differences of all preceding Cubes so deducted.

Progression by 6.

The Progression by 6 is all one in event with the total multiplied by 6, and so is 3 times the square, and 3 times the root united: for 3 times 400 square to 20, and 3 times 20 make 1260.

How to find out any sum of totals of Roots.

In the next place I shall evidence how any progressive sum of totals of Roots may be found out: as suppose it be required to give the number of 35 totals of Roots.

In this and the like case, ever take the next following root and multiply into a Cube, then deduct the root, and divide the residue by 6: so 36 multiplied into a Cube makes 46656, and 36 taken out it is 46620, which divided by 6 gives 7770 total of totals to 35 root the last total is 630, and that doubled makes 1260: so here are 3 remarkable numbers near each other 666, a total 1260 a square and root united, and 7770 a total of totals to 35 root.

How many times a total is included in its Cube.

Moreover, any total of roots is included as

many

many more times in its Cube, as its next precedent root is, and leaves its root as a remainder: so divide 46656 by 70, its double foregoing root, it leaves 36 remainder, and gives 666.

How to find out any total of even or odd Squares.

Now to manifest some use of this Progression, hereby is found the number of all even Squares, or of all odd Squares: for as 1540 is total of totals to 20 root, so also it is total of the first 10 even Squares, and 1330 total of totals, to 19 root is total to the first 10 odd Squares, both which united make the perfect sum of the 20 first Squares, but there is a little readier way to know the sum of any number of Squares which will ensue in the next Page.

A way of finding any number of Squares.

So that as 2 totals of roots next each other, alwaies make a Square, so 2 totals of such total makes the sum of all Squares.

Moreover, divide any total of totals to an even root, and it will give the sum of all Squares, to half so many roots; so divide 1540 by 4, it gives 385, which is total to the first 10 Squares; so that 4 times the sum of all Squares, is total of totals to a root as big again as its own; so 4 times 1 the first Square is total of totals, to 2 the second root, and 4 times 5 the 2 first Squares, makes 20 total of totals to 4 root, *&c.*

What proportion a total is of a Square.

Again observe, that as every total is framed

of half a root and half a Square; ſo every total of totals conſiſts of half the ſum of all Squares, and of half the totals: ſo 2870 total of 20 Squares, being middled is 1435 and half 210 is 105, which added make 1540.

Again, 6 times the total of totals alwaies makes the Cube to the root following, all but its root, ſo 6 times 1540 makes 9240, 21 wants. Further having any ſum of odd totals, to any even root as 1540, to find out which are the even totals, or which odd.

Firſt, Mediate the ſame it is 770 now to even totals; look the ſum of half ſo many roots which is 55 which added to 770, make 825, or take 55 out of 770, it gives 715. *Et Sem.*

Small eaſe to Extracting Root Square.

All the eaſe that I can offer to the extracting of root Square, is, that if it be an even ſquare it may be divided by 4, and the product will ſhew half its root: if again by 4 a quarter, if again by 4 half a quarter. But every odd ſquare will be divided but once by 4, and gives the whole Square and root to its half root: or which is all one, it gives the ſquare to its half Root with $\frac{1}{4}$, ſo 361 ſquare to 19 divided by 90 $\frac{1}{4}$ which is 81, and 9 $\frac{1}{4}$ ſquare to 9 $\frac{1}{2}$.

The readieſt way to find any number of Squares.

Now the readieſt way to find the ſum of all ſquares is as followeth:

If the ſum of the 10 firſt ſquares be required, multi-

multiply 10 unto a Square, it is 100. And again, 100 by 10 is 1000, then join the Cube and Square and Total: it makes 1155, which divide by 3, and it gives 385 total to 10 Squares. so for 20 join 8009 the Cube 400 that Square, and 210 the total it makes 8610, and divided by 3 gives 2870 total to 20 Squares.

Some use of the knowledge of the total of Squares.

Now to demonstrate some use of finding out the sum of all Squares; imagin the last Square at bottom, and then each preceding lying on each other to one Square at top it sheweth the contents of any such Square mount, but the steps are but narrow: but the sum of all even or odd gives broader steps, and but half the ascent.

The usual way to extract root Square.

Before any further proceeding can well be, it is requisite to hold forth the usual way to extract Root Square: prick the last Figure toward the right hand, ~~and~~ and so every other figure, and look how many pricks shall be so many digits, doth the root consist of as in 144 it consists of 2, now take out alwaies the greatest Square under the last prickwhich is but 1 in this; and set it in the quotieut, then double it, having first deducted one, and see 144 (1 how oft it is, contained in the next Figure, which is twice set down 2 144 (12.) then take it out of 4 twice, and nothing remains; lastly, you must multiply the last Figure found again

into

into a Square, and ſo 2 times 2 takes out all: ſo in 1296 the biggeſt ſquare in 12 is 9, the Root of which is 3: ſet 3 in the quotient, and take 9 out of 12, and 3 remains, then double 3 it is 6, then ſee how many times 6 is in 39 which is ſix; now take out 36, and 36 remains, and multiply 6 again into a ſquare, and it takes out all:

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 2
√1296 ( 3
  x 3
√1296 ( 36
  x 5
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So in this Square: 9 is the biggeſt Square in 11, ſet 3 in the quotient, then double that 3 it is 6, then ſee how many times is in 29, it is contained 4 times, then ſay 4 times 6 is 24; take 24 out of 29 remains 5025, out of which the Square of 4 muſt be taken, which is 16, which taken out of 5025. and it leaves 345, then again double the 2 Figures in the quotient, *viz.* 34 makes 68; now ſet 68 under 345, it is contained 5 times in it, and leaves 25 which the Square of 5 taketh out, and ſo 345 is the root to 119025.

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√√9025 (34
  x 6
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How to value any Fraction remaining in Root Square.

If you Extract the Root of any Square and a Fraction remain, that is the numerator, and the root doubled muſt be the denominator to it.

To know difference of Squares.

If any would know how much any Square of any Root differs from its next precedent Square or Subſequent: as how much the Square of

666 differs from the Square of 667 roots: in this and the like Queſtions do but add both roots together, and that is the juſt difference, *viz.* 1333.

To know how many Roots are included in any Total.

And if you would know how many Roots are included in any total of numbers, as in 666, double the total and then take the Square, and if it be an even total, the quotient and remainder will be both alike, as in this 666 doubled makes 1332 the Root Square of which is 36, and ſo alſo the remainder 36; ſo in any number taken promiſcuouſly as 648 double it makes 1296 whoſe ſquare Root being 36 and nothing remaining, ſheweth that it is total to one Root leſs, *viz.* 35 whoſe total it is 630, and doubled 1260. If any Fraction do remain, it is ſo many Unites towards the next total.

Some ſmall eaſe in Multiplying into Squares.

As to eaſe in Multiplying into Squares, if it be to be multiplied by $\frac{1}{2}$, and a whole number as $4\frac{1}{2}$, by $4\frac{1}{2}$ and the like to the whole number Multiplied into it ſelf, ever add the whole root as to 16 add 4, ſo it is $20\frac{1}{4}$; if by $4\frac{1}{4}$ add half the root it makes 18; if by $4\frac{3}{4}$ to 16 add the root and half, ſo it makes 22.

How to Multiply into one leſs than a juſt Square.

Now if you would multiply a Root into

one leſs then its Squar

whic

Square into one less, which may be of great use in searching out the Diagonal Line, that is, the Line leading from one corner of a Square to another over-thwart, which ever makes a Square as big again as the sides do. In this case the exactest way in great Square is tedious; to instance in a small number, 12 times 12 makes 144, now 144 doubled is 288, and that is one less than a just Square, for 17 times 17 makes 289, therefore 16 is the biggest whole Root, and 16 times 16 makes but 256; now the Fraction to be added is 33 one above double to the Root, and 34 the denominator which is the next Root doubled; but for to save that labour, evermore the double Root added will complear the sum, and the numerator will be but one to the denominator 1156 in that cited instance.

How to find out imperfect Cubes to the exactest imperfect Squares.

Now to proceed to a Cube it being a Root Multiplied into a Square, and then that Square multiplied by the said Root, there are no more whole Cubes than there are whole Roots; as also there are no more whole Squares; but now supposing every intervening number betwixt Square and Square, to be an imperfect Square with the least Fraction imaginable, as 2 or 3 to be a Square, or 5, 6, 7, 8, or 10, 11, 12, 13, 14, &c.

&c. if it be required to know the Cube to each, being ſuppoſed to be a Cubick Square. Multiply the ſum given into a Cube and the Root Square of that Cube is the exacteſt Cube to it; as if it be demanded what is the Cube to 15, being accompted a Cubick Square: 15 multiplied into a Cube, makes 3375, whoſe Square Root is $58\frac{11}{15}$, and that is Cube to 15 Square.

This may more eaſily be conceived by obſerving thoſe numbers that have perfect Squares in them, as 64, 729, 4696, which are Cubes to 4, 9, 16, as the Square Root of each of theſe Cubes is a Root in its place; to inſtance:

If the Square 4 be a Root, 8 is the Cubes Root Square; if the Square 9 be a Root, 27 is Root Square to 729; if 16 Square be a Root, 64 is the Square Root of 4096. So if 10 Square be a Root, the Square Root of 1000, which is 31.39 is its Cube, and ſo for the reſt.

And for the preſenting this more obvious to mean capacities, as alſo for eaſe in extracting Roots of perfect Cubes, a Table of 40 Roots Squares, and Cubes doth enſue, whereby any one that hath no skill in the uſual way of extracting Cubick Root by Diviſion may find out any Cubick Root, not exceeding 240, and enlarging the Table to 166 Cubes, may find any Root quickly not exceeding 1000.

A TABLE of Squares.

Roots.	Squares.	Cubes.	Roots.	Squares.	Cubes.
1	1	1	21	441	9261
2	4	8	22	484	10648
3	9	27	23	529	12167
4	16	64	24	576	13824
5	25	125	25	625	15625
6	36	216	26	676	17576
7	49	343	27	729	19683
8	64	512	28	784	21952
9	81	729	29	841	24389
10	100	1000	30	900	27000
11	121	1331	31	961	29791
12	144	1728	32	1024	32768
13	169	2197	33	1089	35937
14	196	2744	34	1156	39304
15	225	3375	35	1225	42875
16	256	4096	36	1296	46656
17	289	4913	37	1369	50653
18	324	5832	38	1444	54872
19	361	6859	39	1521	59319
20	400	8000	40	1600	64000

Now in this Table, as if 4 be a Root, 8 is the Root Square of its Cube: so if 5 be a Square, the Root Square of 125 is its Cube; and as every Square being a Root, its Cube is the Root Square of such Cube as followeth in order if 9 Square be a Root, 27 is latent in 729: So 10 as before being a Cubick Square, its Cube lies hid in 1000, and is its Root Square, *&c.* as it 36 be a Root, 216 is the Root Square of 46656 its Cube, and so on *ad infinitum.*

Moreover, this Table will in 31 Cubes shew very near what is the Root of any Cube, whose Roots exceeds not 1000, for if 961 be a Root, 22791 is the Root Square of its Cube; and if 1024 be a Root, then 32768 is the Root Square of its Cube.

But a Root of 3 Digits is the greatest Example that I have seen used in 3 Authors.

How to give a near guess to a Cubick Root.

Now the way to come nigh the Root, is, the extracting of the Root Square of any Cube, as 216 Root Square of 46656 being found to be a Cube to 36; a Cubick Square sheweth 36 to be its Root exactly: so 64000 its Root Square, is 252, and a Fraction in this case not to be regarded.

Now

Now 252 falling in the Cube Table betwixt the the Cube of 6, and the Cube of 7 sheweth it to be above 36 Root, and not 49, and by a little heed taking a guess, may be given pretty night to the true Root, which in this is 40.

And the extracting of Root Square is much easier than the extracting of the Cubick Root.

Before I proceed how to produce the certain Root only by Division, with the help of a Table, as aforesaid.

How to frame Cubes by Arithmetical Progression, beginning at a Unite.

I shall perform what is before hinted at, how to frame Cubes by Progession Arithmetical, beginning at a Unite: provided they be perfect Cubes arising from a whole Root.

How each Cube is formed by Progression Arithmetical, beginning at its Root, is shewed before.

Now to frame each Cube from a Unite, and so proceeding in odd numbers, or to resolve any Cube into such Progression, the Root Square must be sought for, which is alwaies the number of places of the Progression, and such Root doubled is the first and last Progression conjoyned.

To

To illustrate this in Cubes that have perfect Square Roots, as 64 its Square Root being 8, sheweth that there are 8 places in that Progression, which 8 being doubled sheweth the last Progression to be 15, and the first is alwaies 1; then 16 Multiplied by 4; half the places gives 64 the Cube, as appear here following, 1, 3, 5, 7, 9, 11, 13, 15.

The next of that sort is 729, whose Square Root is 27, which doubled makes 54: so 53 is the last Progression, and 1 the first, and 54 taken 13 times and a half, or half the number 54 is 27, and so 27 by 27 gives 729 the Cube.

As this holds in all Cubes that have perfect Root Square: so also doth it in all other Cubes, but Fractions will arise, and a little use will soon bring to a readiness in valuing thereof for the denominator, must alwaies be the Roat doubled.

I need not repeat the manner thereof, but refer back to another place.

Further ease in framing imperfect Cubes.

One thing more would be heed(ed) in framing of Cubes from impetfect Roots, as $4\frac{1}{2}$ by $4\frac{1}{2}$, or $4\frac{1}{4}$ by it self, or $4\frac{3}{4}$ in like manner for ease in the work reduce it to whole number: as for $4\frac{1}{2}$ by $4\frac{1}{2}$ take 9 and multiply it into a Cube, and then divide any Cube by 8, it alwaies produceth a Cube, whose Root as little again: so 729 by 8 gives 91 Cube to $4\frac{1}{2}$ Now

Now for 4½, consider doubled, it makes 8½, and that again double makes 17 whose Cube is 4913; now that divided by 8 gives 614⅛, that divided again by 8 gives 76⅝, and a small Fraction; every even Cube divided by 8, gives the Cube to its just half Root; and every odd Cube (if its Root be first deducted, and) it be divided by 8 it gives the Cube Square and Total to half the Root of its foregoing Cube: so 9261 Cube to 21, the Root taken out is 9240 that divided by 8 is 1155 the Cube Square and Total to 10 Root, and the like.

Now though the foregoing Table will lead to a near guess of a Cubick Root; yet it supposeth one to be perfect in extracting of a Root Square.

How the knowledge of extracting Cubick Root may be attained in an easier way with help of a Table.

But now I shall advance to hold forth how any one may attain to the knowledge of any Cubick Root exactly without skill to extract Root Square or Cubick in the usual assured way, and that sooner than an Artist in the usual way can perform the same.

Alwaies provided, that the Table aforesaid will not serve to find out a Cubick Root that exceeds 240, but a Table of 166 Cubes will avail to find out any Root, not exceeding a thousand, and

and such a Table any one that is versed in Multiplication may soon compile.

The way to find out the Cubick Root by the foregoing Table is only by Division by the Digit 6.

Another thing remarkable of the Digit 6.

Before I demonstrate the manner, take notice how remarkable this Digit is in all Cubes: for do but take out the Root of any Cube, and the residue is a number of sixes. And further note, that every sixth Cube consists of an equal sum of sixes; and so every following Cube is augmented in its Root by 1, 2, 3, 4, or 5, till it reach the next 6, and then reiterates that order which no other Digit doth, but only its half the Digit 3.

One Instance of ease in Cubes consisting of Sixes only.

Now to proceed to the manner of finding the Cubick Root by the Table foregoing, or a larger composed Table of Cubes; I shall first shew how it is effected in an even six fold Cube, as 1728, divide this by 6; and again, its product by 6, and the off-come again by six, as in the Example; any may soon learn to divide by 6 without setting the 6 under, by only bearing 6 in mind, and the sum alwaies remaining,

1728
288
48
8

now

now 8 being the laſt Diviſion is a Cube whoſe Root is 2, which ſheweth that the Root of 1728 is juſt 2 ſixes or 12; and ſo it will frame in every Cube that is framed of a Root, which hath an even number of ſixes included therein, that is every ſixth Cube.

How to find out the Root when the Cube doth not conſiſt of even Sixes.

But now to ſhew the like in ſuch Cubes whoſe Roots are otherwiſe, as in the next Cube 2197 firſt divide by 6; and note, the remainder as in this Example; then after divide again by 6, not regarding the Fraction in the 2 next Diviſions, if any be: ſo you ſee the laſt Diviſion gives 10, which being more than 8, and not ſo much as 27 ſheweth that the Cube of 8 included in 10 denotes the number of Sixes, to which the remainder (here but a Unite) added makes 13 the juſt Root to 2197, and ſo I ſhall demonſtrate by Examples of the 4 next Cubes.

2197
366 1
61
10

274

2744
457 2
76
12 So to 2 ſixes add 2 it gives 14 for the Root.

3375
562 3
93
15 Still to 12 add 3 it gives 15 Root

4096
682 4
113
18 Still to 12 and 4 it gives 16 Root.

4913
818 5
136
22 Still add 5 to 12 the Root is 17.

5832
972
162
27

So now it produceth a Cube whoſe Root being 3, ſheweth the Root of 5832 to be 3 ſixes or 18, and ſo all along.

How by view to see the first and last Digits of a Cubick Root.

Moreover, Any one unskilful in Extracting the Roots aforesaid, having learnt to be perfect in the knowledge of the first 9 Cubes: So as to find out the first Digit, only by view with the like inspection, may see the last Digit either in any Cube of 2 or 3 Digits in its Root, for 1, 4, 5, 6, 9 in the Root, do alway terminate alike in the Cube, and 8 Root always ends in two, and 2 Root in 8; also 7 Root in 3. and 3 in 7: So that computing in the Cube, so it is easily observed.

And then in perfect Cubes, so soon as the middle Digit is found in a Cube from 3 Digits in the Root, the whole is found out.

The usual and still needful way of Extracting Cubick Root.

But notwithstandiug all these waies of ease, of Extracting Cubick Root, the usual way is necessary when the Cubick Root is to be extracted out of an imperfect number, and therefore I shall briefly demonstrate the usual manner of proceeding therein, in 2 Examples; First, in 1728 the Cube to 12 Root, and the number of inches in a foot of Timber.

In this Example prick the first number overhead next to the right hand, and then leave ever 2 Digits betwixt, as in this 1728, then see what

Cube

Cube is contained in the laſt prick which here is but 1: ſo ſet 1 in the quotient, as in Example; then take 1 out of it, and there remains 728, then put a Cipher to 1 it is 10, and multiply 10 into a Square it is 100; ſet them down as is directed, then ſet under 3 to each, and multiply by 3, ſo it makes 30 300, which 300 is the deviſor; then ſee how many times 300 is to be found in 728 which is twice: ſo ſet 2 in the quotient, then take twice 300, which is 600, and ſet it down as above, then multiply 30 by the Square of 2, which is 4, it makes 120: ſet that down, then take the Cube of 2 which is 8, ſo all makes 728, and 12 is the Root to 1728; if the firſt Digit had been 2, then it ſhould be 20 and 400, and multiplied by 3, it had been 60, and 1200, and ſo it frameth in the other Digits following, 3, 4, 5, 6, 7, 8, 9.

```
1728 ( 12
10  100
3     3
30  300
 600
 120
   8
```

To inſtance in another Cube 41063625, the biggeſt Cube Root under 41 is 3, ſet 3 in the quotient, take out 3 Cube it is 14; then add a Cipher it makes 30, and its ſquare 900; Then by 2700 divide the remainder from the firſt to the middle prick (which is 14063) by 2700 it gives 4; ſet that

```
14
41063625 (345
14063625
   30  900
    3    3
   90 2700
     10800
      1440
        64
```

by 3 above, then under write 4 times 2700, which makes 10800; then multiply 90 by 16 the Square of 4, and set it under as above, then add the Cube of 4, and it makes 12304, then take out of 14063625 the said 12304, and there remains 1759625, then add a Cipher to 34 it makes 340, and its Square is 115600, each multiplied by 3 make 1020, 346800; now divide 1759625 by 346800 it gives 5 to be added to 34

then multiply 346800 by 5 it gives	1734000
then mult: 1020 by 25 the sq: 5 makes	25500
And the Cube of 5 makes	125
All which added makes———	1759625

and takes out all which sheweth 41063625 to be a perfect Cube, and 345 to be its Root; heedfully observe in dividing to take out not only the Divisor, but the multiplicate by the square of the last Digit, and also the Cube as in the above named instances may appear.

As touching the doubling of a Cube, I never saw but one Author which mentions it, which is in Lattin a small Tract of 81 *Octavo* pages concerning Arithmetick, his name is *Gemma Frisius*, and it was bound up with *Kekerman's* Mathematicks. He mentions a misterious way that *Plato* directed to the finding it out, but saith there was an easie way to resolve it.

It is Kekerman *that writes of it* Page 102.

This I have obſerved of it, That a quarter of a Root added to a Root is the ſame biggeſt whole number of its Root, for as 4 Root Cube is 64, ſo 5 gives 125 Cube, which wants 3 of double 128; and ſo how many fours are included in any Root, ſo many times 3 as the Cube of the Root is in Unites: ſo many is the Root defective, as 8 Cube is 512; double 1024 now 10 Cube wants 8 times 3, or 24 of double, and ſo forwards; but the Root ſquare of the doubled Cube muſt needs be a leſs Cube, whoſe Cubick Square is the Root to the double Cube, as I have before mentioned.

The fore-named Author, *Gemma Frizius* manifeſts ſome uſe of proceeding to Extract higher Roots to the reſolving of ſome Queſtions belonging to the Rule of *Coſſa* or *Algeber*, by the Rule of Falſhood in his 45, 59 *pages*, and hints farther of his deſign to ſhew the extracting of ſuch Roots (as *Maſterſon*'s *Book* mentions) *Page* 57.

As alſo of the Rule of *Algeber*'s being a Divine Rule, *page* 64. then which he never ſaw any thing more excellent amongſt the Mathematical Arts, *page* 39. and there manifeſts his intentions to publiſh a peculiar Tract of it (*Deo Favente*) as alſo in *page* 64. unleſs he was prevented by others.

Whe

The Authors desire.

Whether ever he made good his promise, I never have enquired: but if he did, I suppose it is lockt up in the Latin Tongue; and if it be of so great excellency, I hope and desire some body that hath attained to the knowledge thereof, will render it more capable to be understood by those that are ignorant of the Latin Tongue, to advance to a greater increase of knowledge therein.

FINIS.